TRANSPORTATION AND SPATIAL EQUILIBRIUM MODELS

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Introduction -

The geographic distribution of production, processing-manufacture, and consumption in the United States creates an interstate commodity flow of great complexity. This product movement becomes quite fluid when it occurs in response to an efficient system of intermarket communication, transportation and pricing. Under such conditions the continental United States assumes the essential attributes of a market area for many commodities.[2/] The concept of market area "implies not only a territory within which forces of supply and demand act upon price in such a way that price changes in one part of the area quickly affect prices in another part, but also that prices tend to differ between market places only by the cost of transportation to central locations of utilization."[3/] That product shipments and price patterns do tend to conform to the precepts of market area performance has been rather carefully demonstrated in past research.[4/]

Within a market area individual states or regions assume a competitive relationship. Firms and industries within the respective regions act to maximize

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1/ The authors are indebted to Ernest R. Bentley, formerly research assistant in the Department of Agricultural Economics, Ohio State University, for empirical examples used in equations 1-3. These data were prepared by Mr. Bentley in the course of research contributing to his Master of Science degree.

2/ Market area attributes of course are not limited to the United States, but neither do they occur as frequently as hypotheses find convenient.

3/ Stout, Thomas T., and R. L. Feltner, "Price Relationships in the Market for Slaughter Hogs in Indiana," Indiana Agricultural Experiment Station Research Bulletin 746, Lafayette, June, 1962.

4/ Ibid. Also, see Bredo, William, and A. S. Rojko, "Prices and Milksheds of Northeastern Markets," Northeast Regional Publication Number 9, Massachusetts Agricultural Experiment Station, Amherst, August, 1952.

their comparative regional advantages or minimize their comparative disadvantages. It is in the interest of individual states and regions therefore to understand the nature of their comparative position, recognize the interregional competitive relationships that are relevant, be aware of the economic forces to which they respond, and to incorporate these valid considerations in their anticipation of the future. Transportation and spatial equilibrium models are analytical tools which seek to aid such decision-making processes.

Data Yield of Transportation and Spatial Equilibrium Models -

Both transportation and spatial equilibrium models seek to determine optimum shipment patterns for trade of a product among three or more market places within a market area where the amount of the commodity produced is equal to the amount consumed.[5/] When a solution is derived with a transportation model, surplus- and deficit-producing regions are predetermined and independent of commodity price. An optimum shipment pattern is reached when all destination requirements have been satisfied and total transportation costs have been minimized. The solution yields (1) the total transportation bill and discloses the (2) direction and (3) volume of trade between each possible pair of regions. When the more penetrating spatial equilibrium approach is employed, the identification of surplus and deficit regions and determination of related unit commodity

5/ Additional assumptions beyond those implied in the statement are essential analysis; among them some that are characteristic of perfect competition: (a) product homogeniety, (b) economic man and his aspirations for profit maximization, and (c) freedom from external control; in this case from barriers to free interregional commodity flow. Moreover, (d) transportation costs are independent of volume (and perhaps direction) of shipment, and (e) regional demand can be represented by known demand functions. Finally, (f) shipments may occur only between surplus and deficit regions. All shipments of net surpluses must originate in surplus regions and terminate in deficit regions, and no trans-shipping may occur.

prices are an integral aspect of the solution. The equilibrium relationship discloses not only (1) the transportation bill and (2) direction and (3) volume of trade, but also (4) per capita consumption and (5) price per unit in each region. An optimum pattern is attained when all destination requirements have been fulfilled, total transportation cost has been minimized, and product value-added therefore has been maximized.

Model Construction -

The construction and completion of transportation models can be structured into several phases:

1. Collection of essential data.
2. Determination of regional boundaries and basing points in the market area.
3. Determination of surplus- and deficit-producing regions.
4. Arriving at a first approximation of product flows.
5. Iteration of product flows to derive an optimum shipment pattern.

Spatial equilibrium models use the transportation model as an integral step, but treat consumption as a function of price rather than as a fixed quantity. The completion of spatial equilibrium models therefore follows the same procedure as outlined above but incorporates additional steps:

2.(a) Determination of national and regional demand functions.
2.(b) Determination of a set of product price differentials between regions.
2.(c) Determination of an equilibrium set of regional prices.
2.(d) Estimation of regional consumption.
6. Iteration of steps 2.(b) through 5 to determine an optimum shipment pattern and associated set of regional equilibrium prices and quantities.

Data Requirements -

Data inputs necessary for transportation model analysis include estimates of (1) regional production, (2) regional consumption and (3) transportation costs. Spatial equilibrium analysis requires in addition (4) functional estimates of regional demand relationships.

The collection of data usually involves a survey of published series and research literature accompanied by estimates derived from original data. Production and consumption data frequently are given. Transportation cost functions and price-consumption relationships usually are estimated. Data which will provide functional estimates by regions probably will not be available or may be prohibitively cumbersome. Functions representative of the total market area probably will have to be modified to represent each region.

Determination of Regional Boundaries -

The primary objective of regional demarcation is to divide the market area into meaningful and homogeneous production and/or consumption areas. To be meaningful there must be enough regions so that solutions are not overgeneralized. In general, the more complicated procedure of the spatial equilibrium model in relation to the transportation model restricts the number of regions which may be conveniently handled. Basing points within each region are chosen to represent the specific location from which all shipments into or out of the region terminate or originate. Some attempt at a central location for such basing points should be made, locating centrally in terms of production and/or consumption within the region and not merely in terms of location relative to the regional boundaries. Customarily some city is chosen through which such shipments might realistically occur.

Determination of Surplus-and Deficit-Producing Regions-

Both transportation and spatial equilibrium models are concerned with the allocation of commodity shipments between each possible pair of regions at the lowest total transportation cost. Since production and consumption occur simultaneously in all regions the only commodity considered for shipment into or out of the region is the net difference between the amount produced and the amount consumed. If any processing occurs between production and consumption it is necessary to make appropriate adjustments to production-equivalent or consumption-equivalent weights so that amounts produced and consumed in the total market area remain equal.

Commodity price level is not a consideration in using transportation models. Therefore, regional production and consumption, not being affected by price, are fixed and determination of surplus and deficit regions is a simple procedure. When the spatial equilibrium approach is employed, the identification of surplus- and deficit-producing regions and determination of the related regional prices is an integral part of the procedure, and surplus-and deficit-producing regions may vary with successive iterations. For either model, iterative steps begin with an initial approximation of product flows.

The First Approximation of Optimum Product Flows -

When three or more regions are involved the optimum pattern of shipment between possible pairs of regions is not straightforward. If the investigator has no a priori basis for predicting what the optimum pattern should be, the "Vogel Approximation[6] is a method of establishing a first feasible basis for solving the transportation problem. For example, suppose there are eight regions arranged as surplus- and deficit-producing (exporting and importing) as in Table 1 (generalization to any number of regions is apparent.)

Total exports and imports necessarily are of equal tonnage. Transportation costs are derived from estimating equations for shipments between each possible pair of regions and entered in the table. Distance between regions is usually

6/ This approximating technique was first presented at the Industrial Engineering Quality Control Conference at Milwaukee, Wisconsin, in 1954, and was reported in research by G. G. Judge. The methods and examples used here parallel those of Judge, G. G. and Wallace, T. D., as found in "Spatial Price Equilibrium Analysis of the Livestock Economy," Oklahoma Agricultural Experiment Station Technical Bulletin TB-78, Stillwater, June, 1959.

Table 1: Commodity Surplus and Deficit, by Regions, and Transportation Cost Between Regions, Hypothetical Circumstance.

Exporting Regions (i)	Importing Regions (j)					Total Exports (Surplus) (a_i)
	4	5	6	7	8	
	(Transportation Costs)					(Tons)
1	1	3	4	6	3	50
2	0	5	6	9	2	80
3	8	5	3	2	9	120
Total Imports (Deficit) (b_j)	90	25	35	40	60	250

obtained by consulting maps for feasible commercial routes. The unit cost of shipment from Region 3 to Region 7, for example, is 2.

Each exporting region can ship to any importing region. The problem is one of allocating the surplus in a way that satisfies all deficit region requirements at a minimum total transportation cost (sum of the products of unit cost of shipment times volume for each corresponding shipment.) There are fifteen cells in the table representing the only possible interregional shipments. At most, seven of these 15 possible shipments need to occur. In general this method assures that, if a minimum cost shipment pattern exists, there will be at most m + n - 1 shipments, where m is the number of exporting regions and n is the number of importing regions. 7/

Working with the data provided in Table 1, the "Vogel Approximation" method selects a set of shipments under a system of priorities which attempt to reach the optimum solution without further iteration.

7/ The fifteen possible shipments (x_{ij}) represented by cells in Table 1 are related to each other, to each region's requirements (a_i and b_j) and to the transportation costs (c_{ij}) by a cost equation and a system of 8 equations in the 15 unknown shipments (x_{ij}). x_{ij} represents the shipment of product from region i to region j and the value of x_{ij} represents the level of the shipment. In equation form the transportation model is: (Footnote 7 continued on next page.)

Generally, the wider the choice of cost alternatives available to each region, the lower will be its priority in receiving or distributing the necessary quantity of the commodity. As more of the alternatives available to each region disappear, the higher its priority rises until it becomes imperative that its "problems" be resolved. The general procedure for estimating an optimum under this system is presented in Table 2.

(Footnote 7 continued)

Minimize C, where

$$(0)\quad C = c_{14}\, x_{14} + c_{15}\, x_{15} + c_{16}\, x_{16} + c_{17}\, x_{17} + c_{18}\, x_{18}$$
$$+ c_{24}\, x_{24} + c_{25}\, x_{25} + c_{26}\, x_{26} + c_{27}\, x_{27} + c_{28}\, x_{28}$$
$$+ c_{34}\, x_{34} + c_{35}\, x_{35} + c_{36}\, x_{36} + c_{37}\, x_{37} + c_{38}\, x_{38}$$

subject to

$$(1)\quad x_{14} + x_{15} + x_{16} + x_{17} + x_{18} = a_1 = 50$$
$$(2)\quad x_{24} + x_{25} + x_{26} + x_{27} + x_{28} = a_2 = 80$$
$$(3)\quad x_{34} + x_{35} + x_{36} + x_{37} + x_{38} = a_3 = 120$$
$$(4)\quad x_{14} + x_{24} + x_{34} = b_1 = 90$$
$$(5)\quad x_{15} + x_{25} + x_{35} = b_5 = 25$$
$$(6)\quad x_{16} + x_{26} + x_{36} = b_6 = 35$$
$$(7)\quad x_{17} + x_{27} + x_{37} = b_7 = 40$$
$$(8)\quad x_{18} + x_{28} + x_{38} = b_8 = 60$$

The "Vogel Approximation" is a systematic method of obtaining a first feasible solution to equations (1) - (8). Actually, the equations never need to be specifically stated. The nature of this equation system insures that only $n + m - 1$ (in this case, 7) shipments can be determined since there are only $n + m - 1$ independent equations among the $n + m$ equations in the system. Moreover, the fundamental theorem of linear programming (of which transportation models are a special case) states that the number of non-zero x_{ij}'s (shipments) occurring in the solution will be no greater than $n + m - 1$ (the number of independent equations).

If the number of positive shipments is less than $n + m - 1$, the solution is said to be "degenerate." In such cases it will be necessary to treat one or more zero level shipments as positive shipments in order to have $n + m - 1$ "shipments" being made. The necessity and procedure for this will be discussed.

Table 2: Format, Vogel Approximation of Optimum Shipment Pattern, Hypothetical Circumstance.

Exporting Regions (i)	Importing Regions (j) 4	5	6	7	8	Total Exports (ai)	Row Cost Differences						
1	10 $\underline{1}$	$\underline{3}$	$\underline{4}$	$\underline{6}$	40 $\underline{3}$	50	2	2	2	0	X_4		
2	80 $\underline{0}$	$\underline{5}$	$\underline{6}$	$\underline{9}$	$\underline{2}$	80	2	2	X_2				
3	$\underline{8}$	25 $\underline{5}$	35 $\underline{3}$	40 $\underline{2}$	20 $\underline{9}$	120	1	2	2	2	2	2	2
Total Imports (bj)	90	25	35	40	60	250							

Column Cost Differences

4	5	6	7	8
1	2	1	4	1
1	2	1	X_1	1
7	2	1		6
X_3	2	1		6
	5	3		9
	5	3		X_5
	X_6	3		
		X_7		

(1) Construct Table 2, entering the computed unit transportation cost (underlined) in the upper right-hand corner of each cell, leaving room for additional entries in the cell.

(2) Observe Row 1 (all possible export opportunities for Region 1). Select the two lowest transportation costs in the row ($\underline{1}$ and $\underline{3}$) and enter the positive difference between these two costs (2) at the right of the table under the heading "Row Cost Differences." Do the same for each row and column, entering the column cost differences beneath the table.

(3) Select the largest value that has been attained from this initial determination of row and column cost differences. It does not matter whether the value represents a row or a column cost difference. (In this case, the value is 4.)

(4) Examine the row or column in the table from which this cost difference was drawn, and select the cell with the smallest transportation cost. (In this case, the cell representing shipments from Region 3 to Region 7 displays the lowest transportation cost: 2.) Assign to this cell the maximum shipment that it can receive. The maximum will be either the total needs of the importing region or the total available from the exporting region. Mark out the row or column that has been satisfied, eliminating it from further consideration and enter some symbol of termination after the appropriate row cost difference or below the column cost difference. (in this case, the maximum the cell could receive was the total quantity needed by Region 7. Column 7 therefore is marked out, and X_1 is entered below the column cost difference.) Also, subtract from the total exports or total imports the amount that has been shipped or received. (In this case, the export capacity of Region 3 has been reduced from 120 to 80 units remaining available for shipment.)

(5) Re-determine the row and column cost differences not considering marked out rows and columns. If a column has just been removed from consideration, then all row cost differences are subject to re-examination, and column cost differences would remain unchanged. If a row has been removed from consideration, then remaining row cost differences remain unchanged, but column cost differences must be examined for probable changes.

(6) After all row and column cost differences have been re-established, the procedure from (2) through (5) is repeated until all shipments have been made.

Preparation for Reiteration

In small models the Vogel Approximation often will provide an optimum solution without further iterations. The determination of whether or not a solution (the Vogel Approximation or any other) provides an optimum requires an additional step which also serves as a basis for introducing a new shipment in the first iteration.[8]

The shipment pattern derived from the Vogel Approximation (or from any method of finding a solution) is reconstructed in tabular form, putting the unit cost of transportation for each of the assumed shipments in the upper right hand corner of the corresponding cell. The unit costs for these assumed initial shipments are underlined. For example, consulting the initial shipment pattern in Table 3, region 1 ships to regions 4 and 8 and the unit transportation costs are 1 and 3, respectively. The entire set of shipments are those from region 1 to regions 4 and 8, from region 2 to region 4, and from region 3 to regions 5, 6, 7, and 8.

Table 3: Reconstruction of Vogel Approximation Shipment Pattern, Check for Optimum and Price Differentials, Hypothetical Circumstance.

Exporting Regions (i)	Importing Regions (j)					Total Exports (a_i)	Price Differentials (U_i)
	4	5	6	7	8		
1	$10^{\underline{1}}$	-4^{-1}	-7^{-3}	-10^{-4}	$40^{\underline{3}}$	50	6
2	$80^{\underline{0}}$	-7^{-2}	-10^{-4}	-14^{-5}	0^{2}	80	7
3	-1^{7}	$25^{\underline{5}}$	$35^{\underline{3}}$	$40^{\underline{2}}$	$20^{\underline{9}}$	120	0
Total Imports (b_j)	90	25	35	40	60	250	--
Price Differentials (V_j)	7	5	3	2	9	---	--

8/ For a detailed discussion of the method for finding the optimum solution to a transportation model see Dorfman, Robert, Paul A. Samuelson, and Robert Solow. Linear Programming and Economic Analysis, McGraw-Hill Book Co., Inc., New York, 1958, Chapter 5.

On the basis of this initial shipment pattern and the corresponding unit costs of transportation, it is possible to estimate the added costs or savings which could be attained if some other shipment were to take place. The numbers in the cells of Table 3 for which shipments do not occur give us the information on the added costs or savings of making each of these shipments. The number in the upper right hand corner of each cell for which the initial assumed shipment is zero represents the indirect cost (indicated by a negative value) or savings (positive value) which would be realized if this shipment occurs. The nature of these indirect costs is such that the differences in indirect costs between any ocrresponding elements of a pair of rows or a pair of columns is equal to the difference between unit transportation costs of the assumed shipments involving corresponding elements of the pair of rows or pair of columns. Consider rows (regions) 1 and 3, for example. Both regions ship to region (column) 8. The difference in unit transportation cost between row 3 and 1 is 9 minus 3, or 6, and the indirect costs for row 1 are 6 less than each transportation cost (for assumed shipments) or indirect cost in row 3. The indirect costs for region 1 shipping to regions 5, 6, and 7 are 5-6=-1, 3-6=-3, and 2-6=-4, respectively. The indirect costs for region 3 shipping to region 4 is found by adding the row 3 and row 1 difference to the transportation cost of shipping from 1 to 4, ie. 1+6=7. By working with pairs of columns, the indirect costs for row two can be found. For example since the indirect cost from region 3 to region 4 is 7 and the transportation cost from region 3 to region 5 (one of the assumed shipments) is 5, the difference between the indirect cost of column 1 and 2 is equal to 2. Thus the indirect cost from region 2 to region 5 is 0-2=-2. By this process the indirect cost can be found for each cell.[9]

9/ In the case of degenerate solutions mentioned in footnote 4, it may be impossible to complete the calculation of indirect costs on the basis of known transportation charges for existing shipments. (Footnote 9 contd. on next page.)

When the upper right hand corner of each cell is filled, either with the actual transportation cost or with the computed indirect cost, determination of whether or not the assumed shipment pattern represents an optimum requires one more step. The indirect costs of the cells for which no shipment is assumed must be compared with the direct costs (unit transportation cost) of making that shipment. Unit transportation costs (from Table 2) must be subtracted from indirect costs in Table 3. For example, this difference for region 3 to region 4 is 7-8=-1. These differences are entered in those cells of Table 3 where shipments do not occur. If all of these differences are negative or zero, the assumed shipments represent an optimum and further iterations are unnecessary. If some differences of indirect and direct costs are zero for non-occurring shipments (e.g. row 2, column 8 of Table 3), they are interpreted to mean that an alternate optimum (equal minimum cost) solution exists and that the shipments involving these "zero difference" cells could occur without increasing the total transportation cost. If any one of the cost differences is positive, the indirect cost, i.e. saving, of that shipment is greater than the direct cost of making that shipment. This indicates that an optimum has not been reached and that further iterations are needed to find the optimum shipment pattern. The appearance of positive cost differences in the first iteration is not unusual. The number of iterations necessary to find an optimum solution usually varies directly with the number of regions in the model.

Subsequent Iterations

The iteration process is simple. The method is to introduce new shipments, one at a time, eliminating a shipment of the previous solution for each new ship-

(Footnote 9 continued)

In this event, additional shipments of zero value, accompanied by corresponding transportation charges, must be assigned to strategic cells. These cells are those which will enable us to compute all indirect costs by the procedure outlined. Enough zero value shipments must be assigned to bring the total number of shipments up to $n + m - 1$. It will then be possible to determine all the indirect costs in the table.

<u>ment that is added</u>. The addition of one new shipment and the elimination of one previous shipment permits (1) re-calculation of indirect costs and (2) subtraction of unit transport costs from the re-calculated set of indirect costs. The objective is to obtain differences less than or equal to zero..

The Addition of a Single Shipment

The new shipment to be introduced is the one which has the <u>largest positive</u> cost difference. If the largestpositive cost difference is found in two or more cells, select that cell for which the largest shipment can be made. The process of introducing a new shipment and altering existing shipments must be done in a way that does not violate the rim requirements, i.e. the total amount that is to be shipped out of or into each of the regions. Therefore, whenever a shipment is added, a counterpart shipment must be removed so that rim requirements remain unchanged and the total number of shipments become no greater than n + m - 1.

An **example serves** to illustrate the method of changing shipment patterns. Assume that the Vogel Approximation led to a solution which was not optional, as in Table 4.

Table 4: Non-Optimal Shipment Pattern and Associated Cost Differences for Hypothetical Case.

Export Region	Import Region					Total Exports (a_i) (tons)
	4	5	6	7	8	
1	$10^{\underline{1}}$	-4^{-1}	$35^{\underline{4}}$	-10^{-4}	$5^{\underline{3}}$	50
2	$80^{\underline{0}}$	-7^{-2}	-3^{3}	-14^{-5}	0^{2}	80
3	-1^{7}	$25^{\underline{5}}$	7^{10}	$40^{\underline{2}}$	$55^{\underline{9}}$	120
Total Imports (b_j) (tons)	90	25	35	40	60	250

As before, the underlined numbers in the upper-right corner of some cells are indirect costs of making shipments appearing in those cells and the numbers in the non-shipment cells (the number in the upper right corner is not underlined in non-shipment cells) represent cost differences. The shipment pattern indicated in Table 4 satisfied the rim requirements, but the cost difference for each cell is not zero or negative. The cell for region 3 and 6 (at the intersection of region 3 row and region 6 column) has a cost difference of 10-3 = 7. Since this is the only cell with a positive cost difference, a new shipment pattern including shipment from region 3 to 6 should be determined. To retain 7 shipments one of the shipments in the present pattern must be eliminated. To determine the new pattern of shipments consider what would be the effect on existing shipments if 1 unit of product were shipped from region 3 to region 6. Starting in cell 3-6 construct a closed circuit of line segments in such a way that all changes in direction are right angles. These "corners" may occur only in cells where shipments are positive. This path must begin and end in cell 3-6. It may, but need not, cross over itself. Thus, we start in 3-6 and proceed "north" to cell 1-6 (we cannot turn at cell 2-6). Here we turn a right angle and proceed "east" to cell 1-8, where we turn "south" to 3-8, where we turn "west" to 3-6 and complete the loop. The shipments at the corners of this path are the only ones which will be affected by making shipment 3-6 positive. The effect on each shipment is determined in the following way. After leaving the origin (cell 3-6), the odd numbered turns (first, third, fifth, etc.) in the path will have decreases in shipments while the even numbered (second, fourth, sixth, etc.) turns will have an increase in shipments. Thus if the shipment in cell 3-6 is increased to 1 unit, the shipments in cell 1-6 and cell 3-8 will be reduced by 1 unit and the shipment at the second corner (cell 1-8) will increase by one unit. Such a procedure permits the alteration of shipment patterns without disturbing rim requirements.

If we continue to increase the amount of the shipment in cell 3-6, we see that when this new shipment becomes 35 units, the shipment in cell 1-6 becomes zero. <u>This must occur</u> so that total shipments will remain n + m - 1. Thirty-six units could not be shipped from region 3 to 6 because this would require a negative shipment in cell 1-6. <u>This is not permissable</u>. Four shipments are changed in this iteration. Cell 3-6 goes from 0 to 35 units, cell 1-6 goes from 35 to 0 units, cell 1-8 goes from 5 to 40 units, and cell 3-8 goes from 55 to 20 units. This new pattern of shipments can be recognized as the previously determined optimum one, a fact which can be verified by computing the cost differences for each cell and noting that all are non-positive.

In regard to this process of determining the new shipment pattern it is important to recognize three points: (1) A complete circuit or path as described can always be found which begins and ends in a cell where a new shipment is to be made. Only <u>one</u> path will exist for each non-shipment cell and this path may, but need not, cross-over itself; it will be characterized only by a closed series of right-angle turns. The path may be traversed in either direction. In the example, if we had turned "west" at cell 1-6, we would be unable to find a path back to cell 6-3 unless we retraced some of our steps. (2) The path begins at the cell representing the new shipment and continues alternately subtracting and adding the shipment quantity at successive corners. (3) The amount of the new shipment is equal to the amount of the smallest of the shipments at the "subtraction corners." The shipment at the "smallest subtraction corner" will not be in the new shipment pattern.

<u>Commodity Price Differentials</u>

The optimum shipment pattern that evolves in the final iteration will yield a set of value (or price) differentials (U_i and V_j, Table 3) representing the difference in value of the commodity among regions relative to a region selected as the base region. These price differentials are among the necessary inputs in the estimation of regional consumption in spatial equilibrium analysis. In Table 3, Region 3 was used as the base region.

When a surplus region is chosen as a base region the value differential of each deficit region relative to the base region is equal to indirect costs (number in upper-right corner of each cell) of shipment from the base region to the deficit region. For shipments which actually occur, indirect costs equal per unit transport costs. In Table 3, for example, the product is worth 7, 5, 3, 2, and 9 units more in regions 4, 5, 6, 7, and 8, respectively, than in region 3. Value differentials between the base surplus region and other surplus regions are determined by subtracting the indirect cost for the surplus region to any deficit region from the indirect cost for the base region to the <u>same</u> deficit region. In Table 3, the indirect costs from regions 1, 2, and 3 (base) to region 8 are 3, 2, and 9, respectively. The value differential for surplus region 1 is 9-3 = 6, and for region 2 is 9-2 = 7. The U_i's and V_j's are these value differentials. Value differentials may be positive, negative or zero relative to the base region. For our example, all differentials are positive indicating that product value is greater in all other regions than region 3, the base region.

The value differentials and indirect costs can be determined also by solving a set of m + n - 1 linear equations. These equations are based on the observation that value differences between regions can be no greater than transport costs, but must be this large. Thus for each shipment which occurs we have:

$$V_j - U_i = C_{ij}$$

where V_j and U_i are price differentials and C_{ij} is the per unit transport costs from region i to region j. In our hypothetical example, there are m + n - 1 = 7 of these equations:

$$V_4 - U_1 = 1 = C_{14}$$

$$V_4 - U_2 = 0 = C_{24}$$

$$V_5 - U_3 = 5 = C_{35}$$

$$V_6 - U_3 = 3 = C_{36}$$

$$V_7 - U_3 = 2 = C_{37}$$

$$V_8 - U_3 = 9 = C_{38}$$

$$V_8 - U_1 = 3 = C_{18}$$

Setting the value (U_3) for the base region equal to zero, we get the set of value differentials recorded in Table 3.

The indirect costs (C'_{ij}) for non-shipment cells can be determined from

$$V_j - U_i = C'_{ij}$$

where the value differentials determined above are used. For example, the indirect cost from region 1 to region 6 is $V_6 - U_1 = 3-6=-3 = C'_{16}$. Note that C'_{ij} will always be less than or equal to C_{ij}, the transport costs, in the optimal solution

Demand Functions in Spatial Equilibrium Analysis

As stated at the outset, data requirements for spatial equilibrium analysis beyond those for transportation models include functional estimates of regional demand relationships. Accompanying examples using pork consumption serve to illustrate the estimating procedure through successive iterations.[10/] United States average per capita pork consumption is represented in the equation:

10/ Examples used herein are found in "Econometric Generalizations of the Ohio Hog-Pork Industry in Interregional Competition," Stout, T. T., E. R. Bentley, and F. E. Walker, Ohio Agricultural Experiment Station Research Bulletin No. 950, October, 1963. Also, Journal of Farm Economics, Vol. XLIV, No. 5, December, 1962, pages 1572-6.

(1) $Yc = 106.7864 - .6863X_1 + .2591X_2 - .0109X_3$ in which:

($Ry = .9439$)

Yc = U. S. per capita pork consumption in pounds.
X_1 = U. S. average retail price of pork, cents per pound.
X_2 = U. S. average price of beef (all grades); cents per pound.
X_3 = U. S. per capita disposable income; 1960 dollars.

Parameters for the equation were based upon annual observations of the independent variables over the 11-year period 1950-1960. Regional estimates of pork consumption which are derived for each set of iterations in arriving at a spatial equilibrium solution necessarily incorporate the parameters used in estimating national consumption equation (1), but regional values are substituted for each of the variables.[11/] Regional estimates of per capita pork consumption may then be derived with the equation:

(2) $Yc_i = 106.7864 - .6863(X_{po} - d_{oi}) + .2591(74.2) - .0109X_{3i}$ in which:

Yc_i = per capita consumption in the ith region.
X_{po} = retail price of pork in the base region; cents per pound.
d_{oi} = price differential between the ith region and the base region as stated by the term ($X_{pi} - X_{po}$).
74.2 = U. S. average price of beef (all grades); cents per pound.
X_{3i} = per capita disposable income in the ith region; 1960 dollars.

However, the retail price of pork in the base region (X_{po}) remains unknown and must be determined with the equation:

$$(3) \quad \sum_{i=1}^{n+m} P_i Yc_i = 126.0145 \sum_{i=1}^{n+m} P_i - .6863 X_{po} \sum_{i=1}^{n+m} P_i - .6863 \sum_{i=1}^{n+m} P_i d_{oi} - .0109 \sum_{i=1}^{n+m} P_i X_{3i}$$

in which:

P_i = population in the ith region
126.0145 = 106.7864 (the a value in equation (2)), plus .2591(74.2), the constant value for average price of beef.

Solving for X_{po} in equation (3) is not difficult, but it is time consuming and the opportunities to make small but costly mistakes are numerous. Values for

11/ In the accompanying example, regional values were substituted only for X_1 (pork price) and X_3 (disposable income). The national average was retained for X_2 (beef price) due to regional data limitations.

d_{oi} are represented in the computations of U_i and V_j from a previous spatial equilibrium iteration or from the final solution of a transportation model. In this respect, the transportation model serves as the essential basis from which spatial equilibrium iterations are generated, much as a gasoline starter engine is employed to start a huge diesel engine. Since total pork consumption ($\sum_{i=1}^{n+m} P_i Yc_i$) is given (population x per capita consumption) and is equal to production, and since values are available for d_{oi}, then the only unknown in the equation is the price per pound of pork in the base region (X_{po}), which may therefore be determined by substitution within the equation.

Iterative Procedures in Spatial Equilibrium Analysis

The spatial equilibrium model therefore is initiated through the following sequence: (1) An optimum shipment pattern is derived with a transportation model. (2) The optimum solution yields a set of commodity price differentials related to a base region (though a base region price is not yet determined). (3) The price differentials are employed in equation (3) to derive a base region price. (4) The base region price and differentials are employed in equation (2) to determine per capita consumption in each region. (5) Per capita consumption is multiplied by population in each region. (6) Surplus and deficit regions are determined. (7) A Vogel Approximation is made. (8) Price differentials are obtained from the tableau. (9) Steps (3) through (8) are repeated through successive iterations until the last two iterations yield identical sets of price differentials indicating that an optimum solution has been reached.

Some Limitations of the Analytical Techniques -

Transportation and spatial equilibrium models provide penetrating approximations of macro economic activity. Perhaps their principal contribution rests in their ability to quantify theoretical formulations which, by their very complexity, heretofore had been unquantifiable in the practical perspective.

It is the data, more than the logic, that constitutes the major problem in using transportation and spatial equilibrium models. For example: Programming techniques permit the researcher to deal with problems of increasing complexity, but do the complexities still outdistance the tools? Once optimum trade patterns have been generated with the best of models and the differences between the generalizations of the model and the performance of an industry are evident, the interpretation is not clearcut. The industry may be uneconomic or the model may be naive. Such models, confronted by the complexity they presume to explain may be always subject to the accusation that the conclusions were derived from information much too limited in scope. Moreover, generalizations are compounded upon initial generalizations; demand functions, transportation functions, production and processing locations and capacities, regional estimates incorporating national parameters;all are necessary inputs and all are generalizations.

Perhaps the essential assumptions are unrealistically restrictive. The assumption of product homogeniety, for example, denies that product differentiation exists or is possible. Assumptions of cost minimization and profit maximization seek to optimize in the short-run the actions of industries that usually are geared to profit-maximization in the long-run. What appears to be short-run inefficiency really may reflect a carefully calculated move toward long-run maximization, but the models do not recognize the possibility. The models deal only with net product flows, yet much trans-shipping does occur. Projections of present trends to approximate future conditions ignore the entrepreneurial role in its capacity to implement changes and effect the rate of technological progress as conditions warrant.

Certainly the limitations of transportation and spatial equilibrium models are real and numerous. It is not at all self-evident that these models generate policy alternatives more effectively than can astute industry management. But it seems apparent that such techniques provide a useful supplement in aiding management decisions, particularly in planning for the future. And it seems probable that continued effort will be invested in improving and applying these analytical tools, and that they will play an increasingly important role in quantifying and forecasting the complex economic phenomena that continually present themselves.

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